HARMONIC STUDY OF A DIAMETRICALLY WOUND SYNCHRONOUS MACHINE

Bert HANNON^{1,2}, Peter SERGEANT^{1,2} and Luc DUPRE²

¹Ghent University, Department of Industrial Technology and Construction, B-9000, Ghent, Belgium

¹Ghent University, Department of Electrical Energy, Systems and Automation, B-9000, Ghent, Belgium E-mail: Bert.Hannon@UGent.be

Abstract. A trend towards ever more accurate modeling has led to highly complex analytical models. However, increasing complexity may also imply increasing computational time. This paper focusses on analytical modeling techniques that use Fourier series to express the magnetic scalar or vector potential. By studying the harmonic content and the dependency of different magnetic potential equations, a great reduction of the computational time can be achieved.

Keywords: Analytical modeling, harmonic content, synchronous machine

INTRODUCTION

In the context of a trend towards electrical machines with higher energy efficiency and higher power density, electromagnetical modeling of electrical machines is very important.

The models presented in literature can be categorized as analytical or finite element models. Mostly analytical models are preferred when insight in the machine's physics and/or computational time is important. In an effort to increase the accuracy of these models, the subdomain modeling technique has recently been used by various authors [1]-[5]. This technique divides the machine in a number of subdomains. In these domains a differential equation for the magnetic vector or scalar potential can be solved. The solutions in the different subdomains are written as Fourier series over space and, possibly, over time as well. The equations for the vector potential in the different subdomains are then linked by imposing physical boundary conditions. These boundary conditions define the integration constants, introduced when solving the differential equations.

Although the subdomain modeling technique can be used to accurately take into account the slotting effect [1]-[3] and/or the effect of induced currents [3]-[4], its complexity also implies higher computational time. The computational time of subdomain analytical models is mainly determined by the number of integration constants that have to be calculated. The number of integration constants, in turn, is determined by the machine's geometry and the number of space and time harmonic combinations that have to be taken into account.

This work aims at reducing the amount of integration constants that have to be calculated and thereby at lowering the computational time. This is done by studying the harmonic content of the machine's magnetic field. The study is based on a model that was presented by the authors in [3]. This model was built to compute the magnetic field in slotted Permanent Magnet Synchronous Machines (PMSMs) with a Shielding Cylinder (SC). The latter is a conductive sleeve wrapped around the magnets to reduce the overall rotor losses at high speed operation [6]-[7]. The subdomains are indicated with an index ν . As shown in Figure 1, $\nu = 1$ stands for the magnet subdomain, 2 for the shielding cylinder and 3 for the air gap. Every slot is a separate subdomain, represented by an index $v = 4i$, with *i* the slot number $(i = 1..N)$.

Note that the presented work is limited to diametrically wound machines.

Figure 1. Machine geometry and subdomains

2D ANALYTICAL SUBDOMAIN MODEL OF A SLOTTED PMSM WITH SC

In [3] a cylindrical system (r, φ, z) , fixed to the rotor, is applied to formulate a solution for the magnetic $\frac{1}{2}$

vector potential's (*A*) differential equation in every subdomain. It is assumed that the vector potential only has a *z* component. For every subdomain, this component can be written as a Fourier series over space and time:

(1a)
$$
A_{z}^{(\nu)}(r,\varphi,t) = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \left(U_{k,n}^{(\nu)} f_{k}^{(\nu)}(r) + V_{k,n}^{(\nu)} g_{k}^{(\nu)}(r) \right) e^{j(k\varphi + (k-n)\Omega t + k\theta_{0})}
$$
if $\nu = 1...3$

(1b)
$$
A_z^{(4i)}(r,\varphi,t) = \sum_{n=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} U_{l,n}^{(4i)} f_l(r) e^{-j\left(\frac{l\pi}{\delta}(\varphi-\delta_i)+\left(\frac{l\pi}{\delta}-n\right)\Omega t + \frac{l\pi}{\delta}\theta_0\right)}
$$
if $v = 4i$

Where *k* and *l* are spatial harmonic orders and *n* is the time harmonic order. Ω is the mechanical pulsation of the machine and θ_0 is the initial angular position of the rotor. $U_{k,n}^{(\nu)}$, $V_{k,n}^{(\nu)}$ and $U_{l,n}^{(4i)}$ are the integration constants, introduced when solving the differential equation. Lastly, δ is the slot opening angle and δ_i is the starting angle of the *i*th slot:

$$
(2) \qquad \delta_i = \delta_1 + (i-1)\frac{2\pi}{N}
$$

HARMONIC STUDY

The vector potential equation presented in (1a) regards every harmonic combination with a spatial period equaling a multiple of 2π radians and a period over time which is a multiple of $\frac{2\pi}{\Omega}$ $\frac{2\pi}{6}$ seconds. However, not every of these harmonic combinations will be present in an actual machine. Consequently, their associated integration constants will be zero. In this section, the non-zero harmonic fields are identified and a dependency between the integration constants in different slots is proven. This will reduce the amount of integration constants that have to be calculated, and thereby the computational time.

The full paper will restrict the amount of integration constants that have to be calculated by imposing that the vector potential should be real, by studying the harmonic content of the remanent flux density in the magnets and the current density in the slots and by considering time periodicity of the machine's magnetic field.

CONCLUSION

By studying the source terms and the machine's time periodicity and by imposing that the magnetic vector potential is a real function, a reduction of the integration constants that have to be calculated can be achieved.

Although this work focusses on the subdomain model presented in [3], the findings are applicable in every model that uses a Fourier representation of the magnetic potential (either scalar or vectorial). The presented study only regards diametrically wound machines. However, a lot of PMSM are equipped with concentrated windings. It would thus be interesting to extend the work presented in this paper to such machines in future work.

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